An Adaptive Tracking Controller Using Neural Networks for a Class of Nonlinear Systems

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***Abstract*— A neural-network-based adaptive tracking control scheme is proposed for a class of nonlinear systems in this paper. It is shown that RBF neural networks are used to adaptively learn system uncertainty bounds in the Lyapunov sense, and the outputs of the neural networks are then used as the parameters of the controller to compensate for the effects of system uncertainties. Using this scheme, not only strong robustness with respect to uncertain dynamics and nonlinearities can be obtained, but also the output tracking error between the plant output and the desired reference output can asymptotically converge to zero. A simulation example is performed in support of the proposed neural control scheme.**

***Index Terms*— Adaptive control, asymptotic stability, neural networks, nonlinear systems, robustness, uncertain dynamics.**

I. INTRODUCTION

IN RECENT years, the neural-network-based control technique has represented an alternative method to solve the problems in control engineering. The most useful property of neural networks in control is their ability to approximate arbitrary linear or nonlinear mapping through learning. It is because of the above property that many neural-network-based controllers have been developed for the compensation for the effects of nonlinearities and system uncertainties in control systems so that the system performance such as the stability, convergence, and robustness can be improved.

It can be seen from the recent development of the neural-network-based control systems that, by suitably choosing neural-network structures, training methods, and sufficient past input and output data, the neural networks can be well trained to learn the system forward dynamics to predict the future behavior of the systems for the predictive control and model following control, or to learn the inverse dynamics for inverse control. However, the stability, error convergence, and robustness have not been fully proved for these off-line trained neural-network-based control systems because of the high nonlinearity of the neural networks and the lack of feedback [1]. The recent developments in [2], [6], and [8] using adaptive neural networks for direct adaptive control have made a great progress in view to solve the above problems. For example, asymptotic error convergence can be guaranteed within a boundary layer by using adaptive neural networks. However, the output tracking error between the controlled plant and its desired reference trajectory cannot converge to zero due to the approximation error between the system uncertainties and the outputs of the neural networks.

In this paper, we propose a new neural-network-based robust adaptive tracking control scheme for a class of nonlinear systems based on [5], [9]–[11], [13], and [14]. It is shown that, unlike all other neural-network-based control schemes [1], [2], [6], [8], neural networks are not directly used to learn the system uncertainties, but they are used to adaptively learn the bounds of uncertain dynamics in a compact set. The outputs of the neural networks then adaptively adjust the gain of the sliding mode controller so that the effects of system uncertainties can be eliminated and the output tracking error between the plant output and the desired reference signal can asymptotically converge to zero. Because the adaptive neural learning skill and the sliding mode control technique are combined in this paper, the proposed neural control scheme behaves with strong robustness with respect to unknown dynamics and nonlinearities. It will be further shown that it is convenient to use neural networks to learn some uncertainty bounds which are time-varying functions with high nonlinearities, and then the sliding mode control can be easily implemented.

This paper is organized as follows. In Section II, the design of the sliding mode control using the known uncertainty bounds are briefly reviewed, and two RBF neural networks used to learn the system uncertainty bounds are formulated. In Section III, an RBF neural-network-based adaptive tracking controller is proposed and robustness and error convergence of the closed-loop control system are discussed in detail. In Section IV, a simulation example using a one-link rigid robotic manipulator is performed in support of the proposed control scheme. Section V gives concluding remarks.

II. PROBLEM FORMULATION

In this paper, we focus on the design of a neural-network-based robust adaptive tracking controller for a class of single-input and single-output nonlinear systems whose dynamical equations can be expressed in the following form:

$$\begin{aligned} x^{(n)}(t) + f(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \\ = b(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t))u(t) \end{aligned} \tag{1}$$

where t is the time, $x(t)$ is the output variable, $x^{(i)}$ $(i = 1, \cdots, n)$ is the ith derivative of $x(t)$, $u(t)$ is

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the control input, and $f(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t))$ and $b(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t))$ are unknown nonlinear functions.

For general consideration, the following assumptions are made.

A1) The nonlinear function $f(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t))$ is upper bounded

$$\begin{aligned} &f(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \\ &\quad < f_0(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \end{aligned} \tag{2}$$

where $f_0(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t))$ is a positive function.

A2) The sign of control gain $b(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t))$ is known ($b(\cdot) > 0$), and it is lower bounded

$$\begin{aligned} &b(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \\ &\quad > b_1(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) \end{aligned} \tag{3}$$

where $b_1(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t))$ is a positive function.

Equation (1) can also be expressed as the following state equation:

$$\dot{x} = AX + Bu + F \tag{4}$$

where $X = [x, \dot{x}, \cdots, x^{(n-1)}]^T$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & 1 \\ 0 & 0 & & \cdots & 0 \end{bmatrix}$$
$$B = [0, \cdots, b(X)]^T$$
$$F = [0, \cdots, -f(X)]^T.$$

The desired reference model for the system (4) to follow is given by

$$\dot{x}_m = A_m X_m + B_m r(t) \tag{5}$$

where $X_m = [x_m, \dot{x}_m, \cdots, x_m^{(n-1)}]^T$, A_m and B_m are known constant matrices, and $r(t)$ is the input of the reference model.

Defining the output tracking error $\varepsilon = x - x_m$, and an error vector

$$e(t) = X - X_m = [\varepsilon, \dot{\varepsilon}, \cdots, \varepsilon^{(n-1)}]^T \tag{6}$$

where $\varepsilon^{(k)} = x^{(k)} - x_m^{(k)}, (k = 0, \cdots, n-1)$, the error dynamics can then be obtained by using (4)–(6) as follows:

$$\dot{e} = Ae + (A - A_m)X_m + F - B_m r + Bu. \tag{7}$$

For further analysis, a variable s is defined as follows:

$$s = Ce \tag{8}$$

where $C = [c_1, \cdots, c_n]$ is chosen such that zeros of the polynomial Ce are in the left half of the complex plane. For convenience, we let $c_n > 0$. Usually, s is called the switching plane variable and $Ce = 0$ is called the sliding mode in sliding mode control [3]–[5], [13]–[14].

If $b_1(X)$ and $f_0(X)$ are known, the standard technique of the sliding mode control can be used to design the control input. The results on the design of the sliding mode controller and the analysis of the error convergence can be summarized in the following theorem.

Theorem 1: Consider the error dynamics in (7) for the nonlinear system (1). If the control input is designed such that

$$\begin{aligned} u = -\frac{\text{sign}(s)}{c_n b_1(X)}[&|CAe| + |C(A - A_m)X_m| \\ &+ c_n f_0(X) + |CB_m r|] \end{aligned} \tag{9}$$

then the output tracking error asymptotically converges to zero.

Proof: Defining a Lyapunov function

$$V = \frac{1}{2}s^2 \tag{10}$$

and differentiating V with respect to time, we have

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= s[CAe + C(A - A_m)X_m + CF - CB_m r + CBu] \\ &= sCAe + sC(A - A_m)X_m - sc_n f(X) - sCB_m r \\ &\quad - \frac{b(X)}{b_1(X)}|s|[|CAe| + |C(A - A_m)X_m| \\ &\quad + c_n f_0(X) + |CB_m r|] \\ &\le -\left(\frac{b(X)}{b_1(X)} - 1\right)|s|[|CAe| \\ &\quad + |C(A - A_m)X_m| + |CB_m r|] \\ &\quad - \frac{b(X)}{b_1(X)}c_n|s|(f_0(X) - |f(X)|) \\ &\le -\frac{b(X)}{b_1(X)}c_n|s|(f_0(X) - |f(X)|) \\ &= -\eta|s| < 0 \qquad \text{for } |s| \neq 0 \end{aligned} \tag{11}$$

where

$$\eta = \frac{b(X)}{b_1(X)}c_n(f_0(X) - |f(X)|) > 0. \tag{12}$$

Equation (11) with (12) means that the switching plane variable s reaches the sliding mode $S = Ce = 0$ in a finite time according to Lyapunov stability theory [5], and then the error dynamics satisfies the following differential equation in the sliding mode:

$$c_n\varepsilon^{(n-1)} + \cdots + c_1\varepsilon = 0. \tag{13}$$

Therefore, the output tracking error asymptotically converges to zero.

In this paper, we consider the case that the positive nonlinear functions $b_1(X)$ and $f_0(X)$ are unknown. Now we define

$$k_1(X) = b_1^{-1}(X) \tag{14}$$
$$k_2(X) = f_0(X) \tag{15}$$

and the following two RBF neural networks are used to adaptively learn the uncertain bounds $k_1(X)$ and $k_2(X)$ in (14) and (15), respectively,

$$\bar{k}_1(X, \hat{\theta}_1) = \hat{\theta}_1^T \phi(X) \tag{16}$$
$$\bar{k}_2(X, \hat{\theta}_2) = \hat{\theta}_2^T \psi(X) \tag{17}$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ are the weight vectors of the RBF neural networks in the above, and the vectors $\phi(X) \in R^{n1}$ and $\psi(X) \in R^{n2}$ are Guassian type of functions defined as

$$\phi_i(X) = \exp\left(-\frac{\|X - c_{1i}\|^2}{\sigma_{1i}^2}\right) \qquad i = 1, 2, \cdots, n1 \tag{18}$$
$$\psi_i(x) = \exp\left(-\frac{\|X - c_{2i}\|^2}{\sigma_{2i}^2}\right) \qquad i = 1, 2, \cdots, n2. \tag{19}$$

Remark 1: $c_{1i} \in R^n$, $c_{2i} \in R^n$, $\sigma_{1i} \in R^1$, and $\sigma_{2i} \in R^1$ in (18) and (19) are predetermined, and the local training technique in [15] can be used to choose $c_{i1}, c_{i2}, \sigma_{1i}$ and σ_{2i}. In this case, the adjustable weights $\hat{\theta}_1$ and $\hat{\theta}_2$ appear linearly with respect to the known nonlinear functions $\phi(X)$ and $\psi(X)$, respectively, [6] and [15].

For the further analysis, the following assumptions are made.

A3) Given two arbitrary small positive constants w_1 and w_2 and two continuous functions $k_1(X)$ and $k_2(X)$, defined in (14) and (15), on a compact set Σ, there exist two optimal weight vectors θ_1^* and θ_2^* such that the outputs of the two optimal neural networks with enough nodes satisfy [2], [6]

$$|\varepsilon_1(X)| = \left|\theta_1^{*T}\phi(X) - k_1(X)\right| < w_1 \tag{20}$$
$$|\varepsilon_2(X)| = \left|\theta_2^{*T}\psi(X) - k_2(X)\right| < w_2. \tag{21}$$

A4) The uncertainty bounds $b_1(X)$ and $f_0(X)$ in (2) and (3) also satisfy the following inequalities on the compact set Σ:

$$0 < b_1(X) < \frac{1}{1 - w_1} \tag{22}$$
$$f_0(X) - |f(X)| > w_2. \tag{23}$$

Remark 2: Two Assumptions A3 and A4 are reasonable. Assumption A3 reflects the approximation capability of neural networks, and it has been proved and used by many researchers [1], [2], [6]–[8], [15]. And A4 gives the flexible ranges of $b_1(X)$ and $f_0(X)$ together with Assumptions A1 and A2.

The objective of this paper is to use the RBF neural networks in (16) and (17) to learn the uncertain bounds $k_1(X)$ and $k_2(X)$ in (14) and (15), and the outputs of the neural networks are then used as the parameters of the controller so that the output tracking error between the plant and its reference model can asymptotically converge to zero and strong robustness with respect to uncertain dynamics can be guaranteed.

III. The Design of the Neural-Network-Based Controller

For the design of the neural-network-based controller, the adjustment of the weights, and the analysis of the error convergence, we have the following theorem.

Theorem 2: Consider the error dynamics in (7) with Assumptions A1 to A4. If the control input u is designed such that

$$u = -\left(\hat{\theta}_1^T\phi(X)\right)^2 c_n^{-1}\mathrm{sign}(s)[|CAe| + |C(A - A_m)X_m| + |CB_m r|] - \hat{\theta}_1^T\phi(X)\hat{\theta}_2^T\psi(X)\mathrm{sign}(s) \tag{24}$$

where the weight vectors are adjusted by using the following adaptive mechanisms:

$$\dot{\hat{\theta}}_1 = \eta_1[|CAe| + |C(A - A_m)X_m| + |CB_m r|]|s|\phi(X) \tag{25}$$
$$\dot{\hat{\theta}}_2 = \eta_2 c_n|s|\psi(X) \tag{26}$$

with adaptive gains $\eta_1 > 0$, $\eta_2 > 0$, and initial values of the weights

$$\hat{\theta}_{1k}(0) \geq 0 \quad \text{and} \quad \hat{\theta}_{2j}(0) \geq 0 \quad (k = 1, \cdots, n1; j = 1, \cdots, n2).$$

then the output tracking error asymptotically converges to zero.

Proof: Defining a Lyapunov function

$$V = \frac{1}{2}s^2 + \frac{1}{2}\eta_1^{-1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{1}{2}\eta_2^{-1}\tilde{\theta}_2^{-1}\tilde{\theta}_2 \tag{27}$$

where

$$\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i \qquad i = 1, 2 \tag{28}$$
$$\dot{\tilde{\theta}}_i = -\dot{\hat{\theta}}_i \qquad i = 1, 2. \tag{29}$$

Differentiating V with respect to time, we have

$$\begin{aligned}
\dot{V} &= s\dot{s} - \eta_1^{-1}\tilde{\theta}_1^T\dot{\tilde{\theta}}_1 - \eta_2^{-1}\tilde{\theta}_2^T\dot{\tilde{\theta}}_2 \\
&= s[CAe + C(A - A_m)X_m + CF - CB_m r + CBu] \\
&\quad - \eta_1^{-1}\tilde{\theta}_1^T\dot{\tilde{\theta}}_1 - \eta_2^{-1}\tilde{\theta}_2^T\dot{\tilde{\theta}}_2 \\
&= sCAe + sC(A - A_m)X_m - sCB_m r - sc_n f(X) \\
&\quad - [|CAe| + |C(A - A_m)X_m| + |CB_m r|]b(X)|s| \\
&\quad \times \left(\hat{\theta}_1^T\phi(X)\right)^2 - b(X)c_n|s|\hat{\theta}_1^T\phi(X)\hat{\theta}_2^T\psi(X) \\
&\quad - [|CAe| + |C(A - A_m)X_m| + |CB_m r|]|s| \\
&\quad \times \left(\theta_1^{*T} - \hat{\theta}_1^T\right)\phi(X) - |s|c_n\left(\theta_2^{*T} - \hat{\theta}_2^T\right)\psi(X) \\
&= -[|CAe| + |C(A - A_m)X_m| + |CB_m r|]b(X)|s| \\
&\quad \times \left(\hat{\theta}_1^T\phi(X)\right)^2 + [|CAe| + |C(A - A_m)X_m| \\
&\quad + |CB_m r|]|s|\hat{\theta}_1^T\phi(X) - \left[|s|c_n b(X)\hat{\theta}_1^T\phi(X)\hat{\theta}_2^T\psi(X)\right. \\
&\quad \left. - |s|c_n\hat{\theta}_2^T\psi(X)\right] + [sCAe + sC(A - A_m)X_m \\
&\quad - sCB_m r] - [|CAe| + |C(A - A_m)X_m| \\
&\quad + |CB_m r|]|s|\theta_1^{*T}\phi(X) - \left[sc_n f(X) + \theta_2^{*T}\psi(X)c_n|s|\right]
\end{aligned} \tag{30}$$

The first two terms in (30) can be written as follows:

$$\begin{aligned}&-[|CAe|+|C(A-A_m)X_m|+|CB_mr|]b(X)|s|\big(\hat{\theta}_1^T\phi(X)\big)^2\\&\quad+[|CAe|+|C(A-A_m)X_m|+|CB_mr|]|s|\hat{\theta}_1^T\phi(X)\\&\qquad=\big(-b(X)\hat{\theta}_1^T\phi(X)+1\big)[|CAe|+|C(A-A_m)X_m|\\&\qquad\quad+|CB_mr|]|s|\hat{\theta}_1^T\phi(X).\end{aligned}\tag{31}$$

It is noted that

$$\begin{aligned}&-b(X)\hat{\theta}_1^T\phi(X)+1\\&\quad<-b_1(X)\hat{\theta}_1^T\phi(X)+1\\&\quad=-b_1(X)\big(\hat{\theta}_1^T\phi(X)-k_1(X)\big)\\&\quad=-b_1(X)\big[\hat{\theta}_1^T\phi(X)+\theta_1^{*T}\phi(X)-\theta_1^{*T}\phi(X)-k_1(X)\big]\\&\quad=-b_1(X)\big[\big(\hat{\theta}_1^T-\theta_1^{*T}\big)\phi(X)+\varepsilon_1(X)\big]\\&\quad\le-b_1(X)\big[\big(\hat{\theta}_1^T-\theta_1^{*T}\big)\phi(X)-w_1\big]\\&\quad=-b_1(X)\Bigg[\big(\hat{\theta}_1^T(0)-\theta_1^{*T}\big)\phi(X)\\&\qquad\qquad+\eta_1\int_0^t(|CAe|+|C(A-A_m)X_m|\\&\qquad\qquad+|CB_mr|)|s|\phi(X))\,dt)\phi(X)-w_1\Bigg]\end{aligned}\tag{32}$$

and by suitably choosing $\hat{\theta}_1^T(0)$ in the sense that

$$\big[\big(\hat{\theta}_1^T(0)-\theta_1^{*T}\big)\phi(X)-w_1\big]>0\tag{33}$$

we have

$$\begin{aligned}&-b(X)\hat{\theta}_1^T\phi(X)+1\\&\quad<-b_1(X)\Bigg[\big(\hat{\theta}_1^T(0)-\theta_1^{*T}\big)\phi(X)\\&\qquad\qquad+\eta_1\int_0^t(|CAe|+|C(A-A_m)X_m|\\&\qquad\qquad+|CB_mr|)|s|\phi(X))\,dt)\phi(X)-w_1\Bigg]<0\end{aligned}\tag{34}$$

Then

$$\begin{aligned}&-[|CAe|+|C(A-A_m)X_m|+|CB_mr|]b(X)|s|\big(\hat{\theta}_1^T\phi(X)\big)^2\\&\quad+[|CAe|+|C(A-A_m)X_m|+|CB_mr|]|s|\hat{\theta}_1^T\phi(X)\\&\qquad=\big(-b(X)\hat{\theta}_1^T\phi(X)+1\big)[|CAe|+|C(A-A_m)X_m|\\&\qquad\quad+|CB_mr|]|s|\hat{\theta}_1^T\phi(X)\le0.\end{aligned}\tag{35}$$

Similarly, using (34), the third term in (30) satisfies the following inequality:

$$\begin{aligned}&-|s|c_nb(X)\hat{\theta}_1^T\phi(X)\hat{\theta}_2^T\psi(X)+|s|c_n\hat{\theta}_2^T\psi(X)\\&\quad=\big(-b(X)\hat{\theta}_1^T\phi(X)+1\big)|s|c_n\hat{\theta}_2^T\psi(X)\le0.\end{aligned}\tag{36}$$

Now considering Assumption A4, we have

$$\begin{aligned}\theta_1^{*T}\phi(X)-1&=\theta_1^{*T}\phi(X)-k_1+k_1-1\\&=\varepsilon_1(X)+k_1-1\\&\le k_1-1+w_1\\&=b_1^{-1}(X)-(1-w_1)>0\end{aligned}\tag{37}$$

the fourth and the fifth terms in (30) can then satisfy the following inequality:

$$\begin{aligned}&sCAe+sC(A-A_m)X_m-sCB_mr\\&\quad-[|CAe|+|C(A-A_m)X_m|+|CB_mr|]|s|\theta_1^{*T}\phi(X)\\&\qquad\le-\big(\theta_1^{*T}\phi(X)-1\big)[|CAe|+|C(A-A_m)X_m|\\&\qquad\quad+|CB_mr|]|s|\le0.\end{aligned}\tag{38}$$

Finally, the sixth term in (30) can be expressed as

$$\begin{aligned}&-sc_nf(X)-c_n|s|\theta_2^{*T}\psi(X)\\&\quad\le-|s|c_n\big(\theta_2^{*T}\psi(X)-k_2+k_2+|f(X)|\big)\\&\quad=-|s|c_n(\varepsilon_2(X)+k_2-|f(X)|)\\&\quad\le-|s|c_n(\varepsilon_2(X)+w_2)\\&\quad=-\kappa_1|s|<0\qquad\text{for }|s|\neq0\end{aligned}\tag{39}$$

with

$$\kappa_1=c_n(\varepsilon_2(X)+w_2)>0.\tag{40}$$

Therefore, using (35), (38), and (39) in (40), we have

$$\dot{V}\le-\kappa_1|s|\qquad\text{for }|s|\neq0.\tag{41}$$

Equation (41) means that the switching plane variable $s=Ce$ converges to zero in a finite time according to Lyapunov stability theory [5]. Then, it can be seen from (13) that the output tracking error asymptotically converges to zero in the sliding mode $Ce=0$.

Remark 3: The robustness properties of the neural-network-based adaptive controller in (24) may be summarized as follows. 1) When the output tracking error is large due to the effects of system uncertainties, the outputs of the RBF neural networks are adaptively increased according to the update laws in (25) and (26). The control gain can then be increased to eliminate the effects of uncertain dynamics, and drive the switching plane variable s to the sliding mode. In the sliding mode, the output tracking error asymptotically converges to zero. 2) The error dynamics of the closed-loop system are only determined by the sliding mode parameters and are insensitive to system uncertainties and bounded disturbances in the sliding mode. 3) It can be seen from (24) that the controller does not require any knowledge of the controlled nonlinear system, and only the outputs, tracking error and its derivative are used for the design of the controller though the system has nonlinearities and parameter uncertainties.

Remark 4: It can be seen from the above theorem that the weights of the RBF neural networks are adjusted in Lyapunov sense. Therefore, it is not necessary for the weights of the neural networks to converge to their optimal values, but the values of the weights are adaptively increased until the switching plane variable s converges to zero. Then the weights will become constants to guarantee that the output tracking error asymptotically converges to zero in the sliding mode.

Remark 5: It can be seen from (24) that the neural networks used in this paper are essential to realize the nonlinear adaptive control law because the uncertainty bounds $b_1(X)$ and $f_0(X)$ are unknown nonlinear functions. However, if $b_1(X)$ and $f_0(X)$ are constants, the design of the controller and adaptive laws can be greatly simplified without using neural networks [5].

Remark 6: The sign function $\mathrm{sign}(s)$ is involved in the control signal in (24), and therefore chattering may occur in the control input. Based on the principle of boundary layer control technique in [3]–[5], the control input can be smoothed by using (s/δ') to replace $\mathrm{sign}(s)$ when $|s| < \delta'$, where δ' is a positive number. As shown in [3]–[5], the boundary layer controller offers a continuous approximation to the chattering control input signal inside the boundary layer, and guarantees attractiveness to the boundary layer and ultimate boundedness of the output tracking error to within a neighborhood of the origin depending on δ'. However, the drawback is that the output tracking error can not converge to zero.

IV. A Simulation Example

To illustrate the adaptive neural control scheme proposed in this paper, a simulation example for a one-link rigid robotic manipulator is performed. The dynamic equation of the one-link rigid robotic manipulator is given by [12]

$$ml^2\ddot{q} + d\dot{q} + mlg\cos(q) = u \tag{42}$$

where the link is of length $\mathbf{l}$ and mass $\mathbf{m}$, and $\mathbf{q}$ is the angular position with initial values $q(0) = 0.1$ and $\dot{q}(0) = 0$.

The above dynamical equation can be written as the following state equation:

$$\begin{bmatrix}\dot{q}\\ \ddot{q}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}q\\ \dot{q}\end{bmatrix} + \begin{bmatrix}0\\ -(ml^2)^{-1}d\dot{q} - gl^{-1}\cos(q)\end{bmatrix} + \begin{bmatrix}0\\ (ml^2)^{-1}\end{bmatrix}u. \tag{43}$$

For simplicity, the parameters in (42) are chosen as

$$m = l = d = g = 1.$$

The reference model is defined as

$$\begin{bmatrix}\dot{q}_r\\ \ddot{q}_r\end{bmatrix} = \begin{bmatrix}0 & 1\\ -16 & -8\end{bmatrix}\begin{bmatrix}q_r\\ \dot{q}_r\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}r(t) \tag{44}$$

where $[q_r(0) \quad \dot{q}_r(0)]^T = [0 \quad 0]^T$ and $r(t)$ is a periodic rectangular signal.

The controller u is designed as

$$u = -\left(\hat{\theta}_1^T\phi(X)\right)^2(10|\dot{\varepsilon}| + |16q_r + 8\dot{q}_r| + |r(t)|)\,\mathrm{sign}(s) - \hat{\theta}_1^T\phi(X)\hat{\theta}_2^T\psi(X)\,\mathrm{sign}(s) \tag{45}$$

where

$$X = [q \quad \dot{q}]^T, \quad \varepsilon = q - q_r \quad s = 10\varepsilon + \dot{\varepsilon} \tag{46}$$

$$\phi_i(X) = \psi_i(X) = \exp\left(-\frac{\|X - b_i\|^2}{\sigma_i^2}\right). \tag{47}$$

According to (24) and (25), are updated by

$$\dot{\hat{\theta}}_1 = 5[10|\dot{\varepsilon}| + |16q_r + 8\dot{q}_r| + |r(t)|]|s|\phi(X) \quad \text{with } \eta_1 = 5 \tag{48}$$

$$\dot{\hat{\theta}}_2 = 5|s|\psi(X) \quad \text{with } \eta_2 = 5. \tag{49}$$

The Runge–Kutta method with the sampling interval $\Delta T = 0.01$ s is used to solve the nonlinear differential equation numerically in this simulation. Fig. 1(a)–(c) shows the system output tracking, tracking error, and control input signal where each of two neural networks has 15 neurones and 15 weights. The widths, centers and initial values of the weight vectors of the Guassian functions are chosen as follows:

$$\begin{aligned}&\sigma_i^2 = 0.25\\ &b_i = [j \quad k]^T \quad (j = 0, 1, 2, \text{ and } k = -2, -1, 0, 1, 2)\\ &\hat{\theta}_1(0) = \hat{\theta}_2(0) =\\ &[0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]^T.\end{aligned}$$

It can been seen that good tracking performance has been obtained.

Fig. 2(a)–(c) shows the good system performance where each neural network has only six nodes and six weights, and the corresponding widths, centers and initial values of the weight vector of the Guassian functions are chosen as follows:

$$\begin{aligned}\sigma_i^2 &= 0.36\\ b_i &= [j \quad k]^T \quad (j = 0, 1, \text{ and } k = -1, 0, 1),\\ \hat{\theta}_1(0) = \hat{\theta}_2(0) &= [0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5]^T.\end{aligned}$$

After we compare Fig. 1(a)–(c) with Fig. 2(a)–(c), the following facts have been noted: First, the RBF networks used for the simulation results in Fig. 1(a)–(c) are over-parameterized, and the amplitude of the control signal is relatively high. Second, by properly choosing the widths of Guassian functions of the RBF neural networks, the number of nodes and weights can be greatly reduced, and the amplitude of the control signal can also be reduced as shown in Fig. 2(a)–(c).

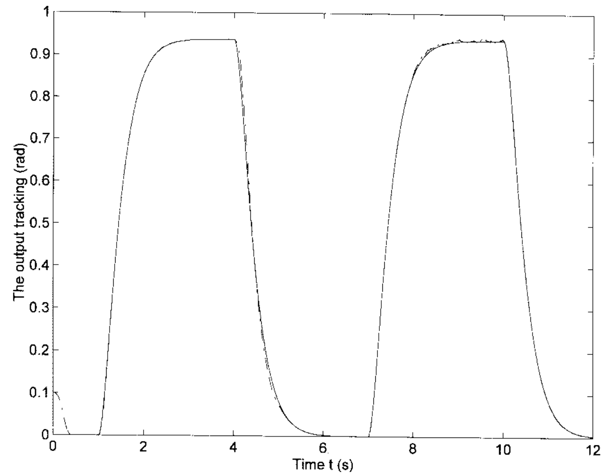
Fig. 3(a)–(c) shows the simulation results where the numbers of nodes and weights of the neural networks are the same as the ones used for the simulation results in Fig. 2(a)–(c). However, the width of the of Guassian functions of the RBF neural networks are modified as $\sigma_i^2 = 0.64$. It is seen that the amplitude of the control signal has been increased and the steady state of the system output is not smooth because of the effects of the large input chattering signal.

Fig. 4(a)–(c) shows the simulation results where the numbers of nodes and weights of the neural networks are the same as the ones used for the simulation results in Figs. 2(a)–(c) and 3(a)–(c), but the initial values of the weight vectors are changed as follows:

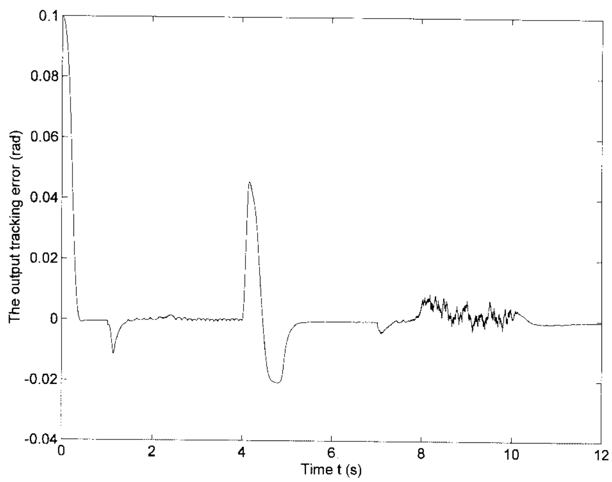
$$\hat{\theta}_1(0) = \hat{\theta}_2(0) = [0.2 \quad 0.2 \quad 0.2 \quad 0.2 \quad 0.2 \quad 0.2]^T.$$

It can be seen that the error convergence is a little bit slower than the one in Fig. 3(a), but the amplitude of the control signal is greatly reduced in the first period and therefore, the steady-state response of the system output has been greatly improved compared with Fig. 3(a).

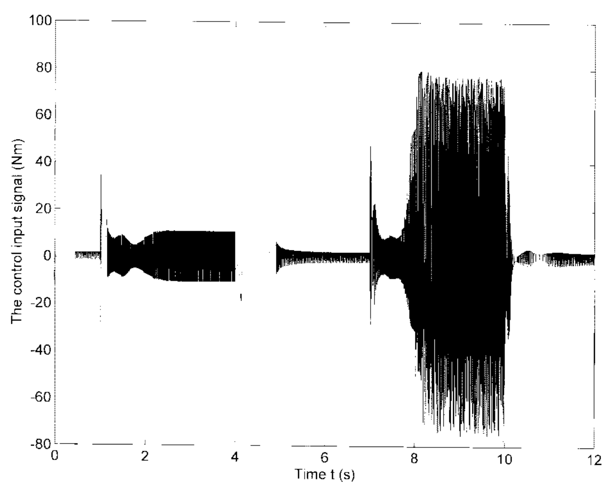
Fig. 5(a)–(c) shows the system performance where the sign function $\mathrm{sign}(s)$ in the control input in Fig. 3(c) is replaced by $s/0.27$. It is seen that the chattering is eliminated and the amplitude of the control signal is greatly reduced [3]–[5]. However, as discussed in Remark 6, the drawback is that the output tracking error cannot converge to zero.



(a)

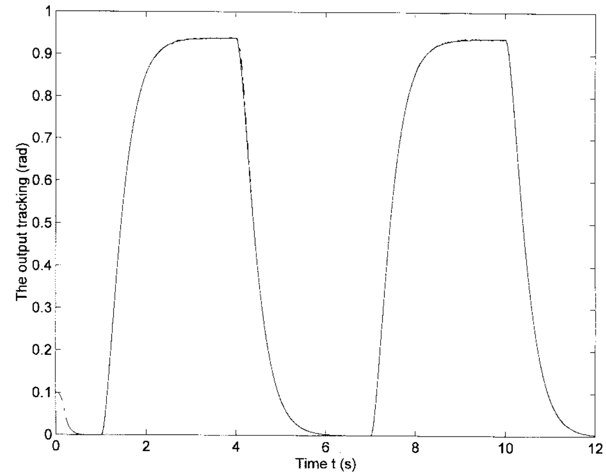


(b)

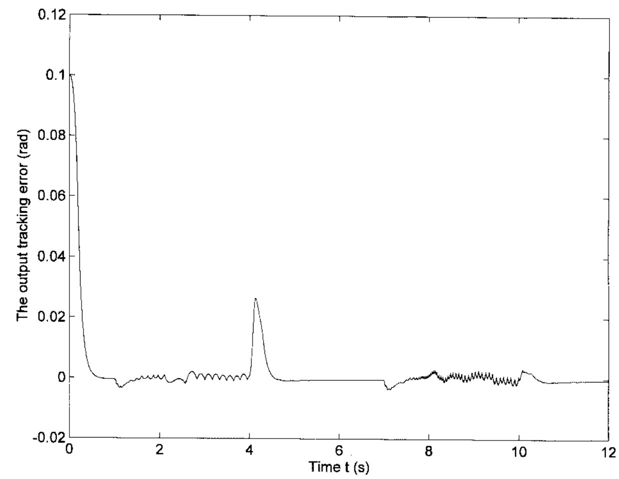


(c)

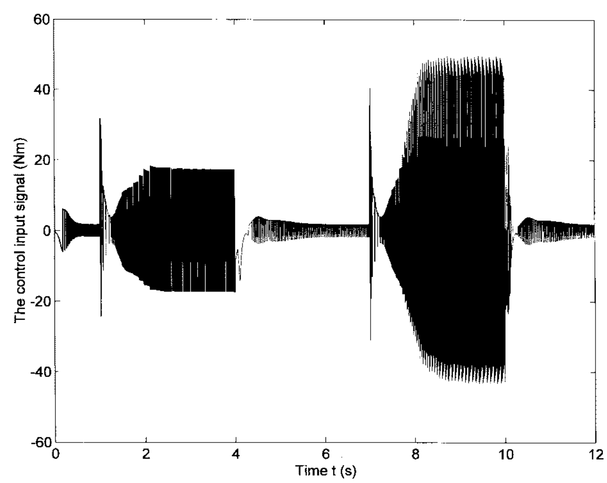
Fig. 1. (a) The output tracking of the angular position. (b) The output tracking error of the angular position. (c) The control input signal.



(a)



(b)



(c)

Fig. 2. (a) The output tracking of the angular position. (b) The output tracking error of the angular position. (c) The control input signal.

V. Conclusion

A new adaptive tracking controller using RBF neural networks is proposed for a class of nonlinear systems in this paper. Our analysis and simulation results have shown that the RBF neural networks can adaptively learn the system uncertainty bounds, and the outputs of the neural networks can

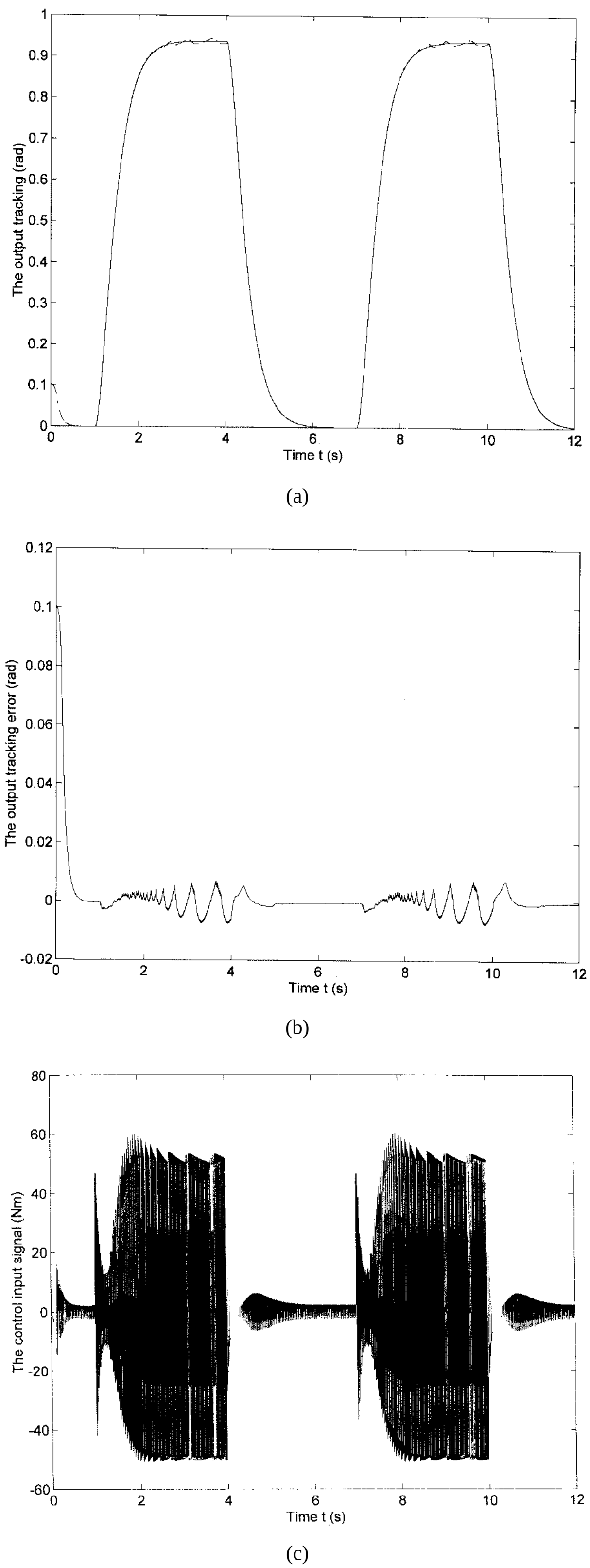


Fig. 3. (a) The output tracking of the angular position. (b) The output tracking error of the angular position. (c) The control input signal.

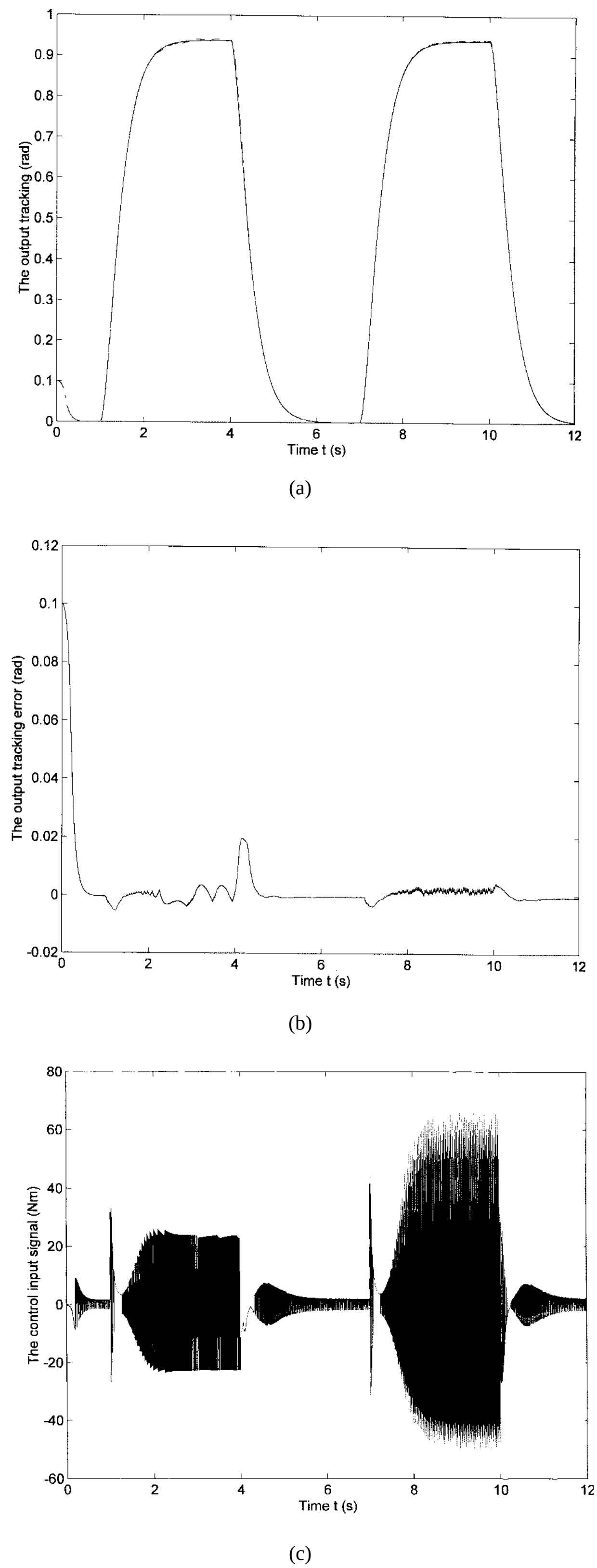
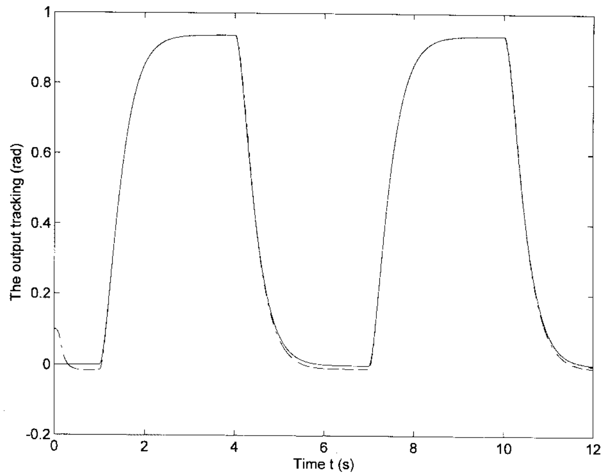
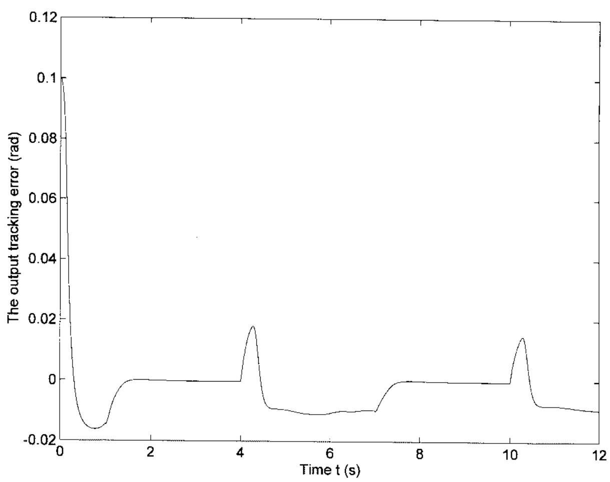


Fig. 4. (a) The output tracking of the angular position. (b) The output tracking error of the angular position. (c) The control input signal.

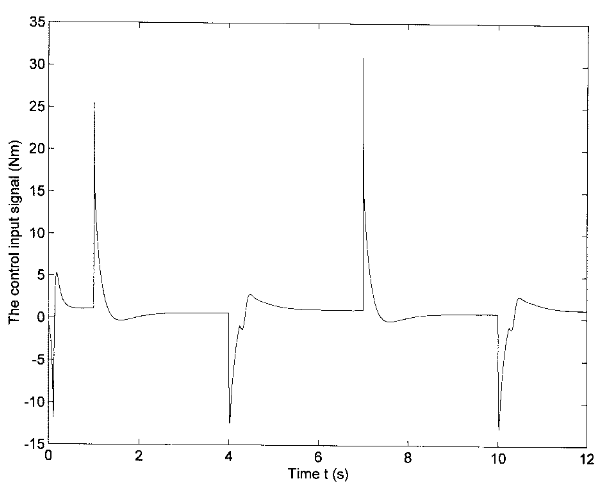
then adaptively adjust the gain of the controller to eliminate the effects of dynamical uncertainties and guarantee asymptotic error convergence. The simulation results have shown the good tracking performance using the proposed control scheme. Also, the effects of the width of the Guassian functions, initial values of the weight vectors, and the number of nodes on



(a)



(b)



(c)

Fig. 5. (a) The output tracking of the angular position. (b) The output tracking error of the angular position. (c) The control input signal.

the system performance have been extensively investigated in the simulation results. The further work on the extension of the proposed neural control scheme to the control of multiinput and multioutput nonlinear systems is under the authors' investigation.

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